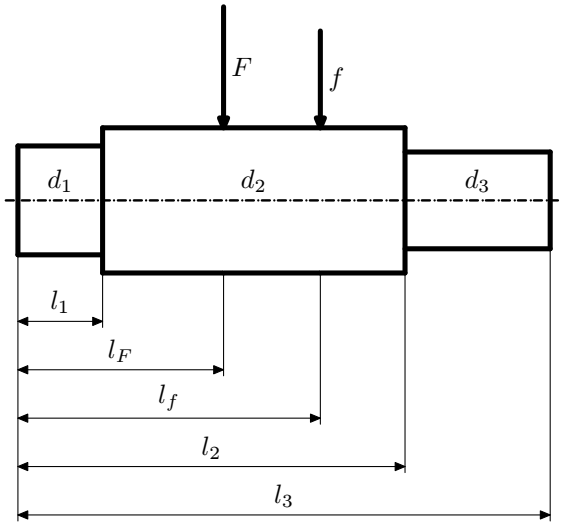
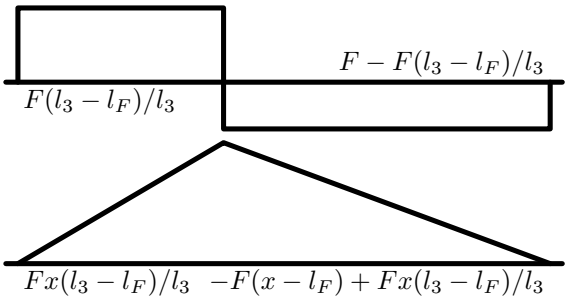
Mohrovy integrály

Výpočet průhybu





Průběh posuvové síly od zatížení silou F

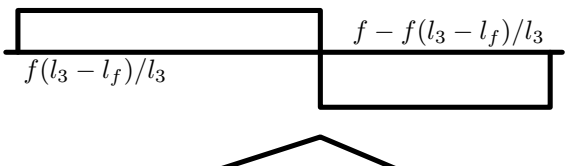
$F(l_3 - l_F)/l_3 \qquad x < l_F$

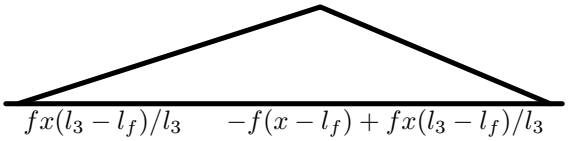
$F - F(l_3 - l_F)/l_3 \qquad x > l_F$

Průběh ohybového momentu od zatížení silou F

$Fx(l_3 - l_F)/l_3 \qquad x < l_F$

$-F(x - l_F) + Fx(l_3 - l_F)/l_3 \qquad x > l_F$





Průběh posuvové síly od zatížení silou f

$f(l_3 - l_f)/l_3 \qquad x < l_f$

$f - f(l_3 - l_f)/l_3 \qquad x > l_f$

Průběh ohybového momentu od zatížení silou f

$fx(l_3 - l_f)/l_3 \qquad x < l_f$

$-f(x - l_f) + fx(l_3 - l_f)/l_3 \qquad x > l_f$

$$u(x = l_f) = \frac{\partial U}{\partial f} = \sum_i \frac{1}{2EI_i} \frac{\partial}{\partial f} \int_{l_{i-1}}^{l_i} \left(M_F(x) + M_f(x)\right)^2 \mathrm{d}x$$

$$u(x = l_f) = \sum_i \frac{1}{2EI_i} \frac{\partial}{\partial f} \int_{l_{i-1}}^{l_i} \left(M_F^2(x) + 2M_F(x)M_f(x) + M_f^2(x)\right) \mathrm{d}x$$

$$\frac{1}{2EI_i} \frac{\partial}{\partial f} \left(M_F^2(x) + 2M_F(x)M_f(x) + M_f^2(x)\right) =$$

$$\frac{1}{2EI_i} \left(0 + 0 + 2M_F(x) \frac{\partial}{\partial f} M_f(x) + 2M_f(x) \frac{\partial}{\partial f} M_f(x)\right)$$

dosadíme $f = 0$

$$\frac{1}{2EI_i}\left(2M_F(x)\frac{\partial}{\partial f}M_f(x)\right) = \frac{M_F(x)\frac{\partial}{\partial f}M_f(x)}{EI_i}$$

$$u(x = l_f) =$$

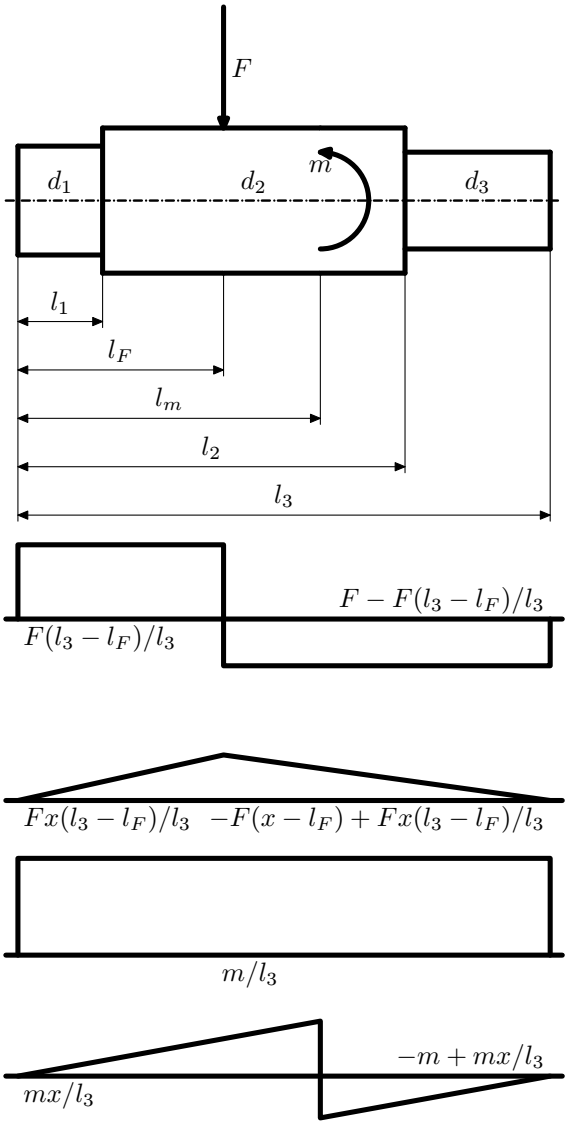
$$\frac{1}{EI_1}\int_0^{l_1}\left[\left(Fx\frac{l_3 - l_F}{l_3}\right)\left(x\frac{l_3 - l_f}{l_3}\right)\right]\mathrm{d}x + \frac{1}{EI_2}\int_{l_1}^{l_F}\left[\left(Fx\frac{l_3 - l_F}{l_3}\right)\left(x\frac{l_3 - l_f}{l_3}\right)\right]\mathrm{d}x$$

$$+\frac{1}{EI_2}\int_{l_F}^{l_f}\left[\left(-F(x - l_F) + Fx\frac{l_3 - l_F}{l_3}\right)\left(x\frac{l_3 - l_f}{l_3}\right)\right]\mathrm{d}x$$

$$+\frac{1}{EI_2}\int_{l_f}^{l_2}\left[\left(-F(x - l_F) + Fx\frac{l_3 - l_F}{l_3}\right)\left(-(x - l_f) + x\frac{l_3 - l_f}{l_3}\right)\right]\mathrm{d}x$$

$$+\frac{1}{EI_3}\int_{l_2}^{l_3}\left[\left(-F(x - l_F) + Fx\frac{l_3 - l_F}{l_3}\right)\left(-(x - l_f) + x\frac{l_3 - l_f}{l_3}\right)\right]\mathrm{d}x$$

Výpočet natočení



Průběh posuvové síly od zatížení silou F

$F(l_3 - l_F)/l_3 \qquad x < l_F$

$F - F(l_3 - l_F)/l_3 \qquad x > l_F$

Průběh ohybového momentu od zatížení silou F

$Fx(l_3 - l_F)/l_3 \qquad x < l_F$

$-F(x - l_F) + Fx(l_3 - l_F)/l_3 \qquad x > l_F$

Průběh posuvové síly od zatížení momentem m

m/l_3

Průběh ohybového momentu od zatížení momentem m

$mx/l_3 \qquad x < l_f$

$-m + mx/l_3 \qquad x > l_f$

$$\varphi(x = l_m) = \frac{\partial U}{\partial m} = \sum_i \frac{1}{2EI_i} \frac{\partial}{\partial m} \int_{l_{i-1}}^{l_i} \left(M_F(x) + M_m(x)\right)^2 \mathrm{d}x$$

$$\varphi(x = l_m) = \sum_i \frac{1}{2EI_i} \frac{\partial}{\partial m} \int_{l_{i-1}}^{l_i} \left(M_F^2(x) + 2M_F(x)M_m(x) + M_m^2(x)\right) \mathrm{d}x$$

$$\frac{1}{2EI_i} \frac{\partial}{\partial m} \left(M_F^2(x) + 2M_F(x)M_m(x) + M_m^2(x)\right) =$$

$$\frac{1}{2EI_i} \left(0 + 0 + 2M_F(x) \frac{\partial}{\partial m} M_m(x) + 2M_m(x) \frac{\partial}{\partial m} M_m(x)\right)$$

dosadíme $m = 0$

$$\frac{1}{2EI_i}\left(2M_F(x)\frac{\partial}{\partial m}M_m(x)\right) = \frac{M_F(x)\frac{\partial}{\partial m}M_m(x)}{EI_i}$$

$$\varphi(x = l_m) =$$

$$\frac{1}{EI_1}\int_0^{l_1}\left[\left(Fx\frac{l_3 - l_F}{l_3}\right)\left(\frac{x}{l_3}\right)\right]\mathrm{d}x + \frac{1}{EI_2}\int_{l_1}^{l_F}\left[\left(Fx\frac{l_3 - l_F}{l_3}\right)\left(\frac{x}{l_3}\right)\right]\mathrm{d}x$$

$$+\frac{1}{EI_2}\int_{l_F}^{l_m}\left[\left(-F(x - l_F) + Fx\frac{l_3 - l_F}{l_3}\right)\left(\frac{x}{l_3}\right)\right]\mathrm{d}x$$

$$+\frac{1}{EI_2}\int_{l_m}^{l_2}\left[\left(-F(x - l_F) + Fx\frac{l_3 - l_F}{l_3}\right)\left(1 - \frac{x}{l_3}\right)\right]\mathrm{d}x$$

$$+\frac{1}{EI_3}\int_{l_2}^{l_3}\left[\left(-F(x - l_F) + Fx\frac{l_3 - l_F}{l_3}\right)\left(1 - \frac{x}{l_3}\right)\right]\mathrm{d}x$$